Homogeneous Function

Scanned with CamScanner

When the variable x and y increased or decreased in a fixed proportion from given values, the corresponding increase or decrease in the function $Z = f(x,y)$ may be greater, in equal, or less proposition. In the very special case $Z = f(x,y)$ increase or decrease always in the same proportion as x and y the function is said to be homogeneous function. of the first degree or to be linear and homogeneous.

A function $f(x,y)$ is homogeneous if

$$f(tx, ty) = t^n f(x,y)$$

t is ~~the~~ any real number and n is the degree of the homogeneous function.

Scanned with CamScanner

$$= \left(\frac{\ }{2} \frac{\ }{Y}\right) Q_1 \quad 2$$

7.4 Application of Euler's Theorem

Euler's theorem states that if the factors of production are paid according to their marginal productivity, the total product will be exhausted. Therefore, in a production function

$$Q = Q(K, L)$$

where K is capital, L is labour and Q is quantity produced, if the factor prices are $\frac{\partial Q}{\partial K}$ and $\frac{\partial Q}{\partial L}$ for capital (K) and labour (L) respectively, then

$$Q = \frac{\partial Q}{\partial K} \cdot K + \frac{\partial Q}{\partial L} \cdot L \tag{7.21}$$

The condition of product exhaustion shown in equation (7.21) is the core of Euler's theorem. The Euler's theorem is applicable to both Cobb-Douglas (C-D) and Constant Elasticity of Substitution (CES) production functions when both of them are linearly homogeneous or when both of them assume the operation of constant returns to scale. Let us first take the case of C-D production function which considers output (Q) as a function of two factors—capital (K) and labour (L) such that

$$Q = AK^{\alpha} L^{\beta} \tag{7.22}$$

where A, α, β are parameters and are positive.

Scanned with CamScanner

The C-D production function given by (7.22) is a homogeneous production function of degree $\alpha + \beta^*$. It is linearly homogenous when $\alpha + \beta = 1$. Assuming linearly condition, (7.22) can be rewritten as

$$Q = AK^{\alpha}L^{1-\alpha} \quad (\text{since} \quad \alpha + \beta = 1) \tag{7.23}$$

To verify whether the C-D function in the form (7.23) satisfies Euler's theorem given by (7.21), we take partial derivatives of Q will respect to K and L. Now

$$\frac{\partial Q}{\partial K} = \alpha AK^{\alpha-1}L^{1-\alpha}$$

or $$\frac{\partial Q}{\partial K} = \frac{\alpha AK^{\alpha}L^{1-\alpha}}{K} = \alpha\frac{Q}{K}.$$

Similarly,

$$\frac{\partial Q}{\partial L} = (1-\alpha)AK^{\alpha}L^{(1-\alpha)-1}$$

or $$\frac{\partial Q}{\partial L} = \frac{(1-\alpha)AK^{\alpha}L^{1-\alpha}}{L} = (1-\alpha)\frac{Q}{L}.$$

Now

$$\frac{\partial Q}{\partial K}\cdot K + \frac{\partial Q}{\partial L}\cdot L = \alpha\tfrac{Q}{K}\cdot K + (1-\alpha)\tfrac{Q}{L}\cdot L$$
$$= \alpha Q + (1-\alpha)Q = Q.$$

Hence C-D production function satisfies the Euler's theorem when $(\alpha + \beta) = 1$.

Scanned with CamScanner